21-241 – Homework assignment week #3

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Reminder

Homework will be given on Fridays and due on the next Friday before 5pm, to me in class or in Andrew Zucker's mailbox in Wean Hall 6113. Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me and Andrew Zucker can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

- Sec. 1.3 : pp. 34–41.

Exercises (18 pts)

1. Prove whether the following are vector spaces or not.[1] (**4*2 pts**)
   - In $\mathbb{R}^3$ with the usual addition and scalar multiplication, the set of solutions of $2x + y - z = 5$.
   - The set of 3×3 real valued matrices such that the sum of the diagonal[2] terms is zero, endowed with the usual addition and scalar multiplication.
   - $\left\{ \begin{bmatrix} x \\ y \end{bmatrix} \;\middle|\; x, y \in \mathbb{R}, y \leq x \right\}$ endowed with usual vector adition and scalar multiplication.
   - In $\mathbb{R}^3$, the intersection between a plane passing through the origin and a line passing through the origin.
2. What is in $\mathbb{R}^3$ the equation of the plane parallel to the one with equation

$$2x + y - 3z = 0$$

   and passing through the point with coordinates $(4, 2, 1)$? Justify your answer. (**3 pts**)
3. Let $a, b, c \in \mathbb{R}$. Find (compute) a linear combination of $\begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$ equal to $\begin{bmatrix} a \\ b \\ c \end{bmatrix}$. What can you deduce in terms of spanning sets ? Can you find another solution ? (**4+2+1 pts**)

[1] Remember that the easiest way to prove that something is a vector space is to prove that it is a subspace of some already known vector space.

[2] The *diagonal* of a matrix will always be implicitely from top-left to bottom-right.