21-268 – Homework assignment week #1

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Reminder

Homework will be given on Wednesdays and due on the next Wednesday before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Reading

1. The syllabus of the class.
2. Blackboard: Michael Hutchings, *Introduction to mathematical arguments*.
   Read this handout to familiarize yourself with some notions of mathematical reasoning and proofs that you might not be familiar with yet. It will help you out for the exercises below.

Exercises (20 pts)

1. (2 pts) Let $S = \{x \in \mathbb{E} : \sqrt{x-2} < x - 4\}$. Express S as an interval or a union of intervals. Prove that your answer is correct.
2. a) (2 pts) Let $S_1, S_2, \ldots, S_m$ be a finite number of open sets in $\mathbb{E}^n$. Prove that $\bigcap_{i=1}^{m} S_i$ is open.
   b) (2 pts) Let S_i be an open set in $\mathbb{E}^n$ for each positive integer, i. Either prove that $\bigcap_{i=1}^{\infty} S_i$ is open or give an example where it is not open.
3. (5*1.5 pts) Consider the function defined by
$$f(x,y) = \frac{x^2 y}{x^4 + y^2}$$
   on $D = \{(x,y) \in \mathbb{E}^2 : x^4 + y^2 \neq 0\}$. Graph the level curves $\{(x,y) \in D : f(x,y) = C\}$ for
$$C = 0,\ \frac{1}{2},\ \frac{-1}{2},\ \frac{1}{2}\sqrt{2},\ \frac{-1}{2\sqrt{2}}.$$
   Plotting points is not sufficient, you should algebraically simplify the equation $f(x,y) = C$ and identify the curves.
4. For each of the following functions defined on $\mathbb{E}^2 \setminus \{(0,0)\}$ determine if the function has a limit as $(x,y) \to (0,0)$. In each case show that your answer is correct: if the limit exists, prove this using the definition of limit and if not prove this using the "two curve method" introduced in class.
   a) (1.5 pts) $f(x,y) = \dfrac{3x^2y^2}{x^2+y^2}$
   b) (1.5 pts) $f(x,y) = \dfrac{xy(x^2-y^2)}{x^4+y^4}$
   c) (1.5 pts) $f(x,y) = \dfrac{x^3y^4}{x^6+y^6}$
   d) (2 pts) $f(x,y) = \dfrac{x^2y^3}{x^4+y^6}$.

   Remark: later on, we will see the notion of *polar change of coordinates* which will considerably simplify this kind of analysis.